Electrical and Electronics Engineering
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Master Semester 2

Course
Smart grids technologies

Stochastic Optimal Power Flow problem

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Outline

Quantifiers in constraints

Robust problems with separable constraints

Stochastic OPF

Robust OPF

Quantifiers in constraints

So far, we have considered OPF problems where **the boundary conditions (e.g. power absorbed/injected by the loads and non-controllable renewables) are known by the operator in a deterministic way**.

However, within an operational context (for instance, in the computation of a day-ahead dispatch problem), **this is not the case since power absorbed by loads and injected by non-controllable renewables (e.g., photovoltaic and wind power plants) is uncertain**.

Therefore, we need to cast the OPF as an **optimization method capable of handling boundary conditions whose realizations are statistically known and expected to fall within intervals provided by forecasting tools.**

Quantifiers in constraints

Consider the following optimization problem

$$(P_1) \min_{u_1, u_2} \varphi(u_1, u_2)$$
$$s.t.$$
$$\text{Constraints } C(u_1, u_2)$$
$$\forall d_1 \; s.t. \; 4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10$$

The variable d_1 **can be considered as the uncertainty of a parameter of the problem** that has a known probability to be within bounds.

It is worth noting that:

- the optimization variables are (u_1, u_2) and **not** (u_1, u_2, d_1);
- we are interested in finding an optimal (u_1^*, u_2^*);
- d_1 is a **dummy variable**.

Quantifiers in constraints

The constraint on the dummy variable

$$\forall d_1 \; s.t. 4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10$$

Is equivalent to:

1. $u_1 + 2\ u_2 \leq \min_{d_1:\ 4 \leq d_1 \leq 5} (10 + 3\ d_1\ )$
2. $u_1 + 2\, u_2 \leq 22$
3. Both
4. None
5. I don't know

Quantifiers in constraints

The constraint on the dummy variable

$$\forall d_1 \; s.t. \, 4 \le d_1 \le 5, \qquad u_1 + 2u_2 - 3d_1 \le 10$$

Is equivalent to:

1. $u_1 + 2\;u_2 \;\le \min_{d_1:\,4\le\, d_1\,\le 5} (10 + 3\;d_1\;)$
2. $u_1 + 2\,u_2 \le 22$
3. Both
4. None
5. I don't know

Answer 3

Answer 1 is true because saying that $\mathrm{Expr} \le f(d_1)$ for all $d_1 \in D$ is the same as saying that $\mathrm{Expr} \le \min_{d_1 \in D} f(d_1)$ whenever Expr does not depend on d_1. Here, $\mathrm{Expr} = u_1 + 2u_2, f(d_1) = 10 + 3d_1$ and $D = [4{,}5]$

Answer 2 is true because $\min_{d_1:\,4\le d_1\le 5} (10 + 3\,d_1) = 22$ and (P_1) is eq. to (P'_1):

$$(P_1{}')\min_{u_1,u_2} \varphi(u_1, u_2)$$

$s.t.$

$\text{Constraints } C(u_1, u_2)$

$u_1 + 2\,u_2 \le 22$

Quantifiers in constraints

Consider a different optimization problem

$$
\begin{aligned}
&(P_2) \min_{u_1,u_2} \varphi(u_1, u_2) \\
&s.t. \\
&\text{Constraints } C(u_1, u_2) \\
&\exists d_1 s.t. 4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10
\end{aligned}
$$

Note that, as in (P_1):

- the optimization variables are (u_1, u_2) and **not** (u_1, u_2, d_1);
- we are interested in finding an optimal (u_1^*, u_2^*);
- d_1 is a **dummy variable**.

Quantifiers in constraints

The constraint on the dummy variable

$$\exists d_1 s.t. 4 \le d_1 \le 5, \qquad u_1 + 2u_2 - 3d_1 \le 10$$

Is equivalent to:

1. $u_1 + 2u_2 \le \max_{d_1:\, 4 \le d_1 \le 5} (10 + 3d_1)$
2. $u_1 + 2\, u_2 \le 25$
3. Both
4. None
5. I don't know

Quantifiers in constraints

The constraint on the dummy variable

$$\exists d_1 s.t. 4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10$$

Is equivalent to:

1. $u_1 + 2u_2 \leq \max_{d_1:\, 4 \leq d_1 \leq 5} (10 + 3d_1)$
2. $u_1 + 2\, u_2 \leq 25$
3. Both
4. None
5. I don't know

Answer 3

Answer 1 is true because saying that $\text{Expr} \leq f(d_1)$ for some $d_1 \in D$ is the same as saying that $\text{Expr} \leq \max_{d_1 \in D} f(d_1)$ whenever Expr does not depend on d_1. Here, $\text{Expr} = u_1 + 2u_2, f(d_1) = 10 + 3d_1$ and $D = [4,5]$

Answer 2 is true because $\max_{d_1:4 \leq d_1 \leq 5}(10 + 3d_1) = 25$. Therefore, (P_2) is eq. to (P'_2)

$$(P_2') \min_{u_1, u_2} \varphi(u_1, u_2)$$

$s.t.$

Constraints $C(u_1, u_2)$

$u_1 + 2\, u_2 \leq 25$

Quantifiers in constraints

Consider a different optimization problem (without quantifiers in constraints)

$$
\begin{aligned}
&(P_3) \min_{u_1,u_2,d_1} \varphi(u_1,u_2) \\
&s.t. \\
&\text{Constraints } C(u_1,u_2) \\
&4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10
\end{aligned}
$$

Note that, differently from (P_1):

- the optimization variables are (u_1, u_2, d_1);
- we are interested in finding an optimal $(u_1^*, u_2^*. d_1^*)$.

Quantifiers in constraints

Solving (P_3) gives the solution of

1. (P_1)
2. (P_2)
3. Both
4. None
5. I don't know

$$(P_1)\min_{u_1,u_2} \varphi(u_1,u_2)$$
$$s.t.$$
$$\text{Constraints } C(u_1,u_2)$$
$$\forall d_1\ s.t.\ 4 \le d_1 \le 5, \qquad u_1 + 2u_2 - 3d_1 \le 10$$

$$(P_2)\min_{u_1,u_2} \varphi(u_1,u_2)$$
$$s.t.$$
$$\text{Constraints } C(u_1,u_2)$$
$$\exists d_1 s.t.\ 4 \le d_1 \le 5, \qquad u_1 + 2u_2 - 3d_1 \le 10$$

$$(P_3)\min_{u_1,u_2,d_1} \varphi(u_1,u_2)$$
$$s.t.$$
$$\text{Constraints } C(u_1,u_2)$$
$$4 \le d_1 \le 5, \qquad u_1 + 2u_2 - 3d_1 \le 10$$

Quantifiers in constraints

Solving (P_3) gives the solution of
1. (P_1)
2. (P_2)
3. Both
4. None
5. I don't know

Answer 2
Indeed, (P_3) solves (P_2) (see next slides).

Solving (P_3) gives an optimal (u_1^*, u_2^*, d_1^*). The corresponding (u_1^*, u_2^*) are an optimal solution of (P_2) and, for this (u_1^*, u_2^*), there exists $d_1 = d_1^*$ that satisfies $4 \leq d_1 \leq 5,\ \ u_1 + 2u_2 - 3d_1 \leq 10$.

$(P_1) \min_{u_1,u_2} \varphi(u_1, u_2)$
$s.t.$
Constraints $C(u_1, u_2)$
$\forall d_1\ s.t.\ 4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10$

$(P_2) \min_{u_1,u_2} \varphi(u_1, u_2)$
$s.t.$
Constraints $C(u_1, u_2)$
$\exists d_1 s.t.\ 4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10$

$(P_3) \min_{u_1,u_2,d_1} \varphi(u_1, u_2)$
$s.t.$
Constraints $C(u_1, u_2)$
$4 \leq d_1 \leq 5, \qquad u_1 + 2u_2 - 3d_1 \leq 10$

Quantifiers in constraints

$$(P_A)\ \min_u \varphi(u)\ \ s.t. \begin{cases} \exists d \in D, C(u,d) \\ C'(u) \end{cases} \qquad (P_B)\ \min_{u,d} \varphi(u)\ \ s.t. \begin{cases} d \in D \\ C(u,d) \\ C'(u) \end{cases}$$

Theorem

1. The optimal values of (P_A) and (P_B) are equal.
2. If $u^*$is an optimal solution of (P_A) then there exists some d^* such that (u^*, d^*) is an optimal solution of (P_B)
3. If (u^*, d^*) is an optimal solution of (P_B), then u^* is an optimal solution of (P_A).

Quantifiers in constraints

Proof

Let us assume the constraints in both (P_A) and (P_A) defining closed sets. This ensures that, when the problems are bounded, the minimum exists and is obtained for each of them.

1) If u is feasible for (P_A), then there exists some d such that (u, d) is feasible for (P_B). Conversely, if (u, d) is feasible for (P_B), then u is feasible for (P_A). Thus (A) feasible ⇔ (B) is feasible.

2) Assume (P_A) is unbounded: for any $M \in \mathbb{R}$, there exists some u, feasible for (P_A), such that $\varphi(u) \leq M$. Since u is feasible for (P_A), there is some d such that (u, d) is feasible for (P_B) and $\varphi(u) \leq M$. Thus, (P_B) is unbounded. Similarly, we have that, if (P_B) is unbounded, then (P_A) is unbounded too. Thus, (P_A) unbounded ⇔ (P_B) is unbounded.
This shows item 1 when the optimal value of (P_A) or (P_B) is infinite.
Assume in the rest that the optimal values of (P_A) and (P_B) are finite. Since the constraints in both (P_A) and (P_B) define closed sets, the minimum exists and is obtained for each problem.

3) Let us prove that if (u^*, d^*) is an optimal solution of (P_B), then u^* is an optimal solution of (P_A) and the optimal values are the same. For all (u, d) that satisfies the constraints of (P_B) we have $\varphi(u) \geq \varphi(u^*)$. Now let u be a feasible value of (P_A); this means that there is one d such that (u, d) satisfies the constraints of (P_B); therefore, $\varphi(u) \geq \varphi(u^*)$. Furthermore, u^* is feasible because there exists some d (take $d = d^*$) such that (u^*, d) satisfies the constraints of (P_B). This proves that u^* is an optimal solution of (P_A). Moreover, the optimal values of (P_B) and (P_A) are both equal to $\varphi(u^*)$.

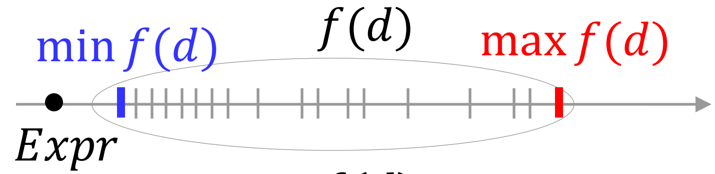
4) Conversely, assume u^* is an optimal solution of (P_A); u^* is feasible for (P_A), i.e. there exists some d^* such that (u^*, d^*) satisfies the constraints of (P_B). We claim that(u^*, d^*) is an optimal solution of (P_B). First, note that (u^*, d^*) is feasible for (P_B). Second, for any feasible point (u, d) of (P_B), we have that u is feasible for (P_A), therefore $\varphi(x) \geq \varphi(x^*)$. This proves that (u^*, d^*) is an optimal solution of (P_B). Moreover, the optimal values of (P_B) and (P_A) are both equal to $\varphi(u^*)$. □

Quantifiers in constraints

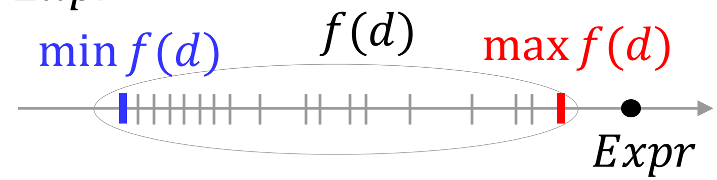
To summarize:

Removal of ∀ and ∃: whenever Expr **does not depend on** d:

$$[\forall d \in D, \text{Expr} \leq f(d)] \Leftrightarrow \left[\text{Expr} \leq \min_{d \in D} f(d)\right]$$

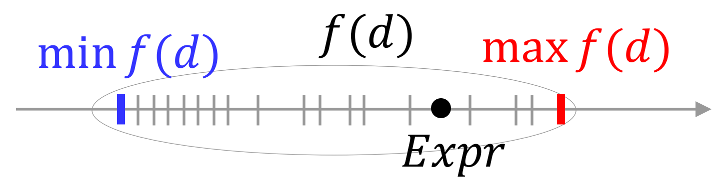


$$[\forall d \in D, \text{Expr} \geq f(d)] \Leftrightarrow \left[\text{Expr} \geq \max_{d \in D} f(d)\right]$$



$$[\exists d \in D, \text{Expr} \leq f(d)] \Leftrightarrow \left[\text{Expr} \leq \max_{d \in D} f(d)\right]$$

$$[\exists d \in D, \text{Expr} \geq f(d)] \Leftrightarrow \left[\text{Expr} \geq \min_{d \in D} f(d)\right]$$



Removal of ∃ with supplementary variables

$$\min_{u} \varphi(u) \ \ s.t. \begin{cases} \exists d \in D, C(u,d) \\ C'(u) \end{cases}$$

can be addressed by solving

$$\min_{u,d} \varphi(u) \ \ s.t. \begin{cases} d \in D \\ C(u,d) \\ C'(u) \end{cases}$$

Quantifiers in constraints

Which problem(s) can be used to solve (P) ?

1. (P_A)
2. (P_B)
3. Both
4. None
5. I don't know

$$(P) \min_{P_{g_2},P_{g_3},P_{l_1},P_{l_2},P_{l_3}} C_1(-P_{g_2}-P_{g_3}+7.00)+C_2(P_{g_2})+C_3(P_{g_3})$$
$$\text{s.t.}\, P_{l_1}\in[0.50;1.50], P_{l_2}\in[0.50;1.50], P_{l_3}\in[4.00;6.00]$$
$$P_{g_1}^{min}\le -P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3}\le P_{g_1}^{max}$$
$$-P_{1,2}^{max}\le 22.2\left(0.036(P_{g_2}-P_{l_2})+0.018(P_{g_3}-P_{l_3})\right)\le P_{1,2}^{max}$$
$$-P_{1,3}^{max}\le 11.1\left(0.018(P_{g_2}-P_{l_2})+0.054(P_{g_3}-P_{l_3})\right)\le P_{1,3}^{max}$$
$$-P_{2,3}^{max}\le 11.1\left(0.018(P_{g_2}-P_{l_2})-0.036(P_{g_3}-P_{l_3})\right)\le P_{2,3}^{max}$$
$$0\le P_{g_2}\le 4$$
$$0\le P_{g_3}\le 4$$

$$(P_A) \min_{P_{g_2},P_{g_3}} C_1(-P_{g_2}-P_{g_3}+7.00)+C_2(P_{g_2})+C_3(P_{g_3})$$
$$s.t.\, \exists P_{l_1}\in[0.50;1.50], \exists P_{l_2}\in[0.50;1.50], \exists P_{l_3}\in[4.00;6.00]$$
$$P_{g_1}^{min}\le -P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3}\le P_{g_1}^{min}$$
...

$$(P_B) \min_{P_{g_2},P_{g_3}} C_1(-P_{g_2}-P_{g_3}+7.00)+C_2(P_{g_2})+C_3(P_{g_3})$$
$$s.t.\, \forall P_{l_1}\in[0.50;1.50], \forall P_{l_2}\in[0.50;1.50], \forall P_{l_3}\in[4.00;6.00]$$
$$P_{g_1}^{min}\le -P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3}\le P_{g_1}^{min}$$
...

Quantifiers in constraints

Which problem(s) can be used to solve (P) ?

1. (P_A)
2. (P_B)
3. Both
4. None
5. I don't know

$$(P) \min_{P_{g_2},P_{g_3},P_{l_1},P_{l_2},P_{l_3}} C_1(-P_{g_2} - P_{g_3} + 7.00) + C_2(P_{g_2}) + C_3(P_{g_3})$$

$$\text{s.t.}\, P_{l_1} \in [0.50; 1.50], P_{l_2} \in [0.50; 1.50], P_{l_3} \in [4.00; 6.00]$$

$$P_{g_1}^{min} \leq -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \leq P_{g_1}^{max}$$

$$-P_{1,2}^{max} \leq 22.2\left(0.036(P_{g_2} - P_{l_2}) + 0.018(P_{g_3} - P_{l_3})\right) \leq P_{1,2}^{max}$$

$$-P_{1,3}^{max} \leq 11.1\left(0.018(P_{g_2} - P_{l_2}) + 0.054(P_{g_3} - P_{l_3})\right) \leq P_{1,3}^{max}$$

$$-P_{2,3}^{max} \leq 11.1\left(0.018(P_{g_2} - P_{l_2}) - 0.036(P_{g_3} - P_{l_3})\right) \leq P_{2,3}^{max}$$

$$0 \leq P_{g_2} \leq 4$$

$$0 \leq P_{g_3} \leq 4$$

Answer 1

(P_B) cannot be used to solve (P) because in (P) we choose the loads P_{l_i} and generators P_{g_i} setpoints whereas in (P_B) we do not know which value P_{l_i} will take.

(P_A) is the same as (P).

$$(P_A) \min_{P_{g_2},P_{g_3}} C_1(-P_{g_2} - P_{g_3} + 7.00) + C_2(P_{g_2}) + C_3(P_{g_3})$$

$$s.t. \exists P_{l_1} \in [0.50; 1.50], \exists P_{l_2} \in [0.50; 1.50], \exists P_{l_3} \in [4.00; 6.00]$$

$$P_{g_1}^{min} \leq -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \leq P_{g_1}^{min}$$

$$\ldots$$

$$(P_B) \min_{P_{g_2},P_{g_3}} C_1(-P_{g_2} - P_{g_3} + 7.00) + C_2(P_{g_2}) + C_3(P_{g_3})$$

$$s.t. \forall P_{l_1} \in [0.50; 1.50], \forall P_{l_2} \in [0.50; 1.50], \forall P_{l_3} \in [4.00; 6.00]$$

$$P_{g_1}^{min} \leq -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \leq P_{g_1}^{min}$$

$$\ldots$$

Outline

Quantifiers in constraints

Robust problems with separable constraints

Stochastic OPF

Robust OPF

Robust problems with separable constraints

In power systems operation and control, we often solve problems like the following (i.e., **robust problems**) that **can be solved by separating the constraints**.

$$(P)\ \min_{u\in\mathbb{R}^n} \varphi(u)$$
$$s.t. \begin{cases} \text{Constraints on } u \\ \left(a^j\right)^T u + \left(b^j\right)^T d \le \gamma_j, \forall d \in \mathbb{R}^p \text{ s.t. } Dd \le c, j = 1{:}J \end{cases}$$

where u (i.e., the decision variable) are generators setpoints and d (i.e., the stochastic disturbance) is the (aggregated) load. Let us **apply the "removal of ∀" to every constraint** $j = 1{:}J$

$$\left(a^j\right)^T u + \left(b^j\right)^T d \le \gamma_j, \forall d \in \mathbb{R}^p \text{ s.t. } Dd \le c$$
$$\Leftrightarrow \left(a^j\right)^T u \le -\left(b^j\right)^T d + \gamma_j, \forall d \in \mathbb{R}^p \text{ s.t. } Dd \le c$$
$$\Leftrightarrow \left(a^j\right)^T u \le \min_{d\in\mathbb{R}^p, Dd\le c} \left[-\left(b^j\right)^T d + \gamma_j\right] \overset{\text{def}}{=} h_j$$
$$\Leftrightarrow \left(a^j\right)^T u \le h_j$$

Robust problems with separable constraints

In power systems operation and control, we often solve problems like the following (i.e., **robust problems**) that **can be solved by separating the constraints**.

$$(P)\ \min_{u\in\mathbb{R}^n} \varphi(u)$$
$$s.t.\begin{cases} \text{Constraints on } u \\ (a^j)^T u + (b^j)^T d \le \gamma_j, \forall d \in \mathbb{R}^p \text{ s.t. } Dd \le c, j = 1{:}J \end{cases}$$

Solution:

1. For every $j = 1{:}J$ solve the problem:

$$(Q_j)\ \min_{d\in\mathbb{R}^p}(-(b^j)^T d + \gamma_j) \text{ s.t.} Dd \le c$$

let h_j be the optimal value of (Q_j)

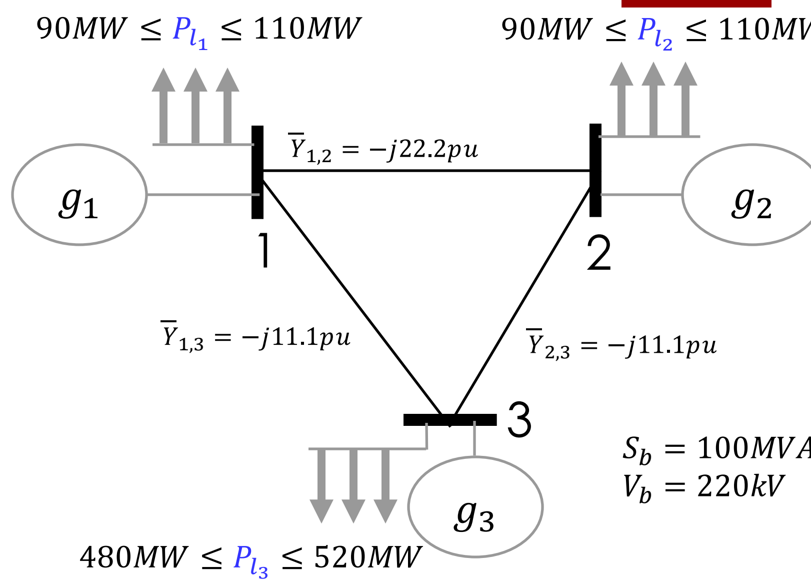
2. Replace (P) by the equivalent problem:

$$(P')\ \min_{u\in\mathbb{R}^n} \varphi(u)$$
$$s.t.\begin{cases} \text{Constraints on } u \\ (a^j)^T u \le h_j,\ j = 1{:}J \end{cases}$$

Robust problems with separable constraints

Example: we would like to compute the usual DC-OPF assuming loads $P_{l_1}, P_{l_2}, P_{l_3}$ (i.e., the disturbances) predicted with some uncertainty.

$$90MW \le P_{l_1} \le 110MW$$
$$90MW \le P_{l_2} \le 110MW$$
$$480MW \le P_{l_3} \le 520MW$$



$S_b = 100MVA$
$V_b = 220kV$

How does this affect the dispatch plan ?

Note that we can control only P_{g_2}, P_{g_3} since bus 1 is the slack bus and P_{g_1} is determined by the power balance of the load flow when $P_{g_2}, P_{g_3}, P_{l_1}, P_{l_2}, P_{l_1}$ are known.

$$\min_{P_{g_2}, P_{g_3}} \sum_{i=1}^{3} C_i(P_{g_i})$$

$s.t.$

$$P_{g_1} + P_{l_1} = 22.2(-\theta_2) + 11.1(-\theta_3)$$
$$P_{g_2} + P_{l_2} = 22.2(\theta_2) + 11.1(\theta_2 - \theta_3)$$
$$P_{g_3} + P_{l_3} = 11.1(\theta_3) + 11.1(\theta_3 - \theta_2)$$

$$P_{g_i}^{min} \le P_{g_i} \le P_{g_i}^{max}, i = 1,2,3$$

$$-P_{1,2}^{max} \le P_{1,2} \le P_{1,2}^{max}$$
$$-P_{1,3}^{max} \le P_{1,3} \le P_{1,3}^{max}$$
$$-P_{2,3}^{max} \le P_{2,3} \le P_{2,3}^{max}$$
$$P_{1,2} = 22.2(0 - \theta_2)$$
$$P_{1,3} = 11.1(0 - \theta_3)$$
$$P_{2,3} = 11.1(\theta_2 - \theta_3)$$
$$-\pi \le \theta_i \le \pi, i = 2,3$$

Quantity	Value
$P_{g_i}^{min}, P_{g_i}^{max}$	$0 \div 400\ MW$
C_1, C_2, C_3	$15, 1, 225\ CHF/MWh$
$S_{12}^{max}, S_{23}^{max}, S_{31}^{max}$	$200, 200, 300\ MW$

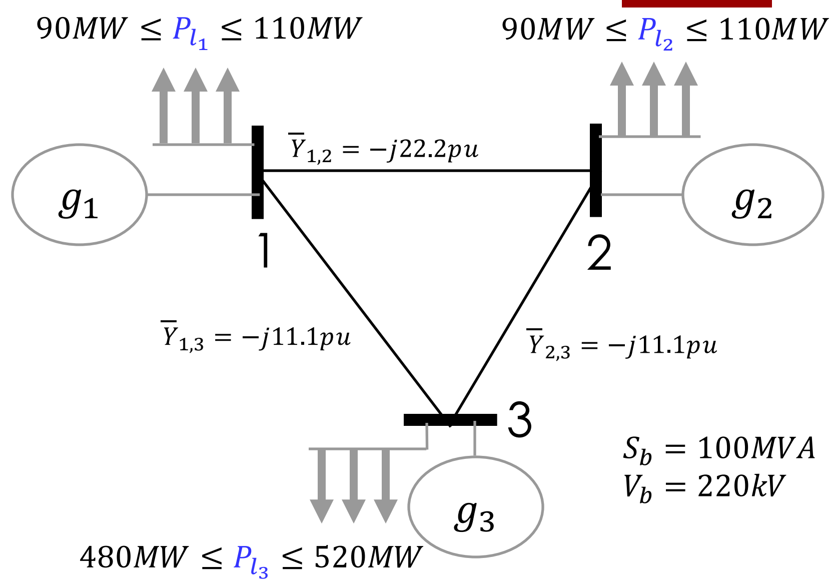
Robust problems with separable constraints

The cost $\sum_{i=1}^{3} C_i\left(P_{g_i}\right)$, is influenced by the disturbance and cannot be known in advance.

Furthermore, the constraints also depend on the disturbance. Our choice on P_{g_2}, P_{g_3} **must work regardless of the disturbance (in this sense must be robust).**

We need to formulate a problem that addresses these two questions:

1. **What should we minimize ?**
2. **Constraints must be adapted to reflect the uncertainty of disturbances.**



$$\min_{P_{g_2}, P_{g_3}} \sum_{i=1}^{3} C_i\left(P_{g_i}\right)$$

$s.t.$

$$P_{g_1} + P_{l_1} = 22.2\,(-\theta_2) + 11.1(-\theta_3)$$
$$P_{g_2} + P_{l_2} = 22.2\,(\theta_2) + 11.1(\theta_2 - \theta_3)$$
$$P_{g_3} + P_{l_3} = 11.1(\theta_3) + 11.1\,(\theta_3 - \theta_2)$$
$$P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max}, i = 1,2,3$$
$$-P_{1,2}^{max} \leq P_{1,2} \leq P_{1,2}^{max}$$
$$-P_{1,3}^{max} \leq P_{1,3} \leq P_{1,3}^{max}$$
$$-P_{2,3}^{max} \leq P_{2,3} \leq P_{2,3}^{max}$$
$$P_{1,2} = 22.2\,(0 - \theta_2)$$
$$P_{1,3} = 11.1(0 - \theta_3)$$
$$P_{2,3} = 11.1(\theta_2 - \theta_3)$$
$$-\pi \leq \theta_i \leq \pi, i = 2,3$$

Quantity	Value
$P_{g_i}^{min}, P_{g_i}^{max}$	$0 \div 400\ MW$
C_1, C_2, C_3	$15, 1, 225\ CHF/MWh$
$S_{12}^{max}, S_{23}^{max}, S_{31}^{max}$	$200, 200, 300\ MW$

Outline

Quantifiers in constraints

Robust problems with separable constraints

Stochastic OPF

Robust OPF

Stochastic OPF

In a stochastic optimisation problem, we need to take decisions (i.e., choose some **control** parameters $u \in U$) **in the presence of a random disturbance** $d \in D$.

The **state** of the controlled system x will be function of both the control and the disturbance $x = F(u, d)$ and must be feasible $(x \in X)$.
As in the previous OPF formulations, the state x consists of all variables in the optimization problem other than u, d (e.g., nodal voltages and branches powers/currents).

The cost is $C(u, x) = C\big(u, F(u, d)\big)$

A **stochastic optimization framework assumes the disturbance to be drawn as a random vector in the set** D (i.e. the modeler **knows the statistical distribution of the disturbance**).

The **objective function is the expected cost** (since the disturbance is randomly drawn) **and the constraints must be satisfied for all possible realizations of the disturbance.**

Stochastic OPF

The **feasibility of the constraints has to be imposed for all the drawn random disturbances** and, therefore, **it is guaranteed with some probability.**

Let $\bar{C}(u) = \mathbb{E}[C(u,x)] = \mathbb{E}_d\big[C\big(u, F(u,d)\big)\big]$

The problem is

$$\min_{u \in U} \bar{C}(u)$$
$$\text{s.t.}\,(\forall d \in D, \exists x, x = F(u,d) \text{ and } x \in X)$$

or, in compact form:

$$\min_{u \in U} \bar{C}(u)$$
$$\text{s.t.}\,(\forall d \in D, F(u,d) \in X)$$

Stochastic OPF

$$\min_{P_{g2},P_{g3}} \; C_1\bar{P}_{g1} + C_2P_{g2} + C_3P_{g3} \qquad \bar{C}(u)$$

$s.t.$

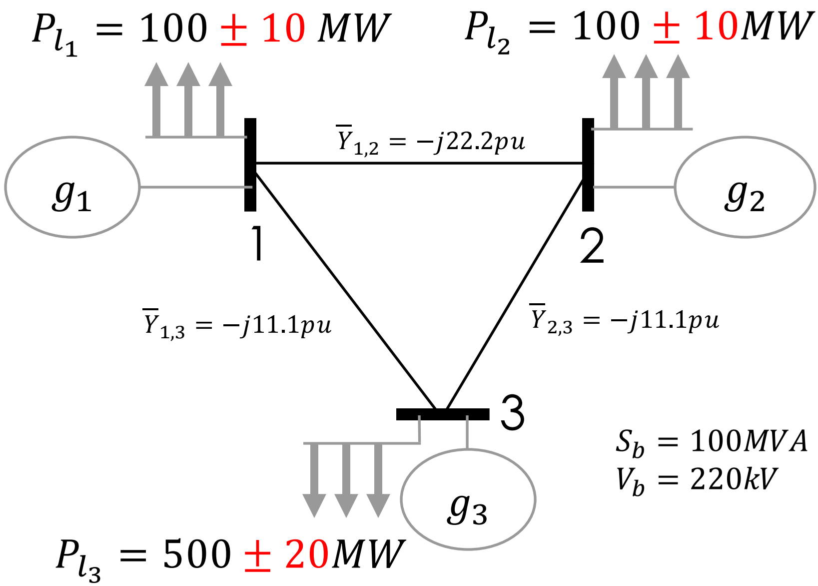
$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max}, i = 2,3 \qquad u \in U$$

$$\forall P_{l_1} \in [0.9,1.1], \forall P_{l_2} \in [0.9,1.1], \forall P_{l_3} \in [4.8,5.2] \qquad \text{for all } d \in D$$

There exist some $P_{g_1}, \theta_2, \theta_3, P_{1,2}, P_{1,3}, P_{2,3}$ *with*

$$P_{g_1} + P_{l_1} = 22.2\,(-\theta_2) + 11.1(-\theta_3)$$
$$P_{g_2} + P_{l_2} = 22.2\,(\theta_2) + 11.1(\theta_2 - \theta_3)$$
$$P_{g_3} + P_{l_3} = 11.1(\theta_3) + 11.1\,(\theta_3 - \theta_2)$$
$$\bar{P}_{g_1} = -P_{g_2} - P_{g_3} + \bar{P}_{l_1} + \bar{P}_{l_2} + \bar{P}_{l_3}$$
$$P_{g_1}^{min} \leq P_{g_1} \leq P_{g_1}^{max}$$
$$-P_{1,2}^{max} \leq P_{1,2} \leq P_{1,2}^{max}$$
$$-P_{1,3}^{max} \leq P_{1,3} \leq P_{1,3}^{max}$$
$$-P_{2,3}^{max} \leq P_{2,3} \leq P_{2,3}^{max}$$
$$P_{1,2} = 22.2\,(0 - \theta_2)$$
$$P_{1,3} = 11.1(0 - \theta_3)$$
$$P_{2,3} = 11.1(\theta_2 - \theta_3)$$
$$-\pi \leq \theta_i \leq \pi, i = 2,3$$

$$x = F(u, d)$$
$$x \in X$$



Decision variables: $u = (P_{g_2}, P_{g_3})$

Disturbance: $d = (P_{l_1}, P_{l_2}, P_{l_3})$

State: $x = (P_{g_1}, \theta_2, \theta_3, P_{1,2}, P_{1,3}, P_{2,3})$

Expected cost: $\bar{C}(P_{g2}, P_{g3}) = C_1\bar{P}_{g1} + C_2P_{g2} + C_3P_{g3}$

with $\bar{P}_{g_1} = -P_{g_2} - P_{g_3} + \bar{P}_{l_1} + \bar{P}_{l_2} + \bar{P}_{l_3}$

we take $\bar{P}_{l_1} = 1\,pu$, $\bar{P}_{l_2} = 1\,pu$ and $\bar{P}_{l_3} = 5\,pu$

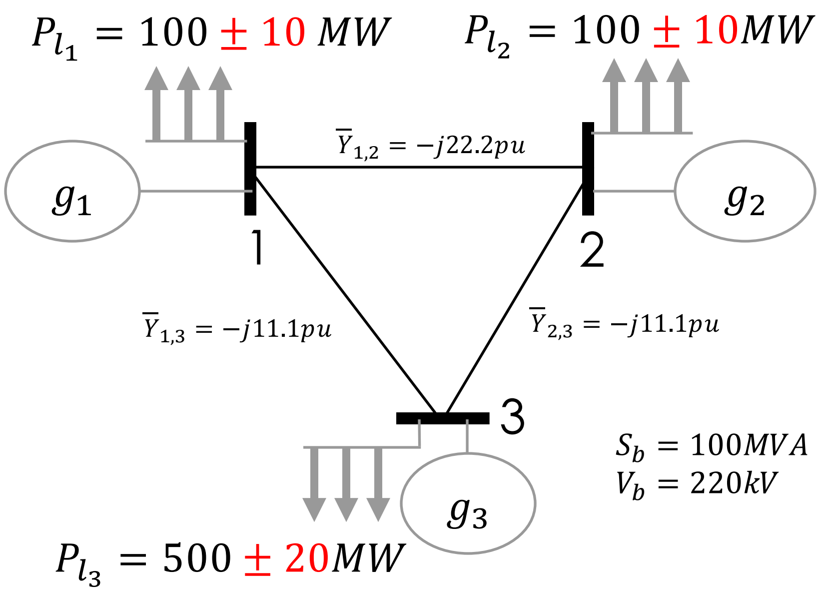
Stochastic OPF

In a compact form we have:

$$\min_{u \in U} \bar{C}(u) \text{ s.t. } (\forall d \in D, \exists x, x = F(u,d) \text{ and } x \in X)$$

or

$$\min_{u \in U} \bar{C}(u) \text{ s.c. } (\forall d \in D, F(u,d) \in X)$$



Decision variables: $u = (P_{g_2}, P_{g_3})$

Disturbance: $d = (P_{l_1}, P_{l_2}, P_{l_3})$

State: $x = (P_{g_1}, \theta_2, \theta_3, P_{1,2}, P_{1,3}, P_{2,3})$

Expected cost: $C_1\bar{P}_{g1} + C_2 P_{g2} + C_3 P_{g3}$

with $\bar{P}_{g_1} = -P_{g_2} - P_{g_3} + \bar{P}_{l_1} + \bar{P}_{l_2} + \bar{P}_{l_3}$

We get

$$\left.\begin{aligned}
P_{g_1} &= -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \\
\theta_2 &= 0.036(P_{g_2} - P_{l_2}) + 0.018(P_{g_3} - P_{l_3}) \\
\theta_3 &= 0.018(P_{g_2} - P_{l_2}) + 0.054(P_{g_3} - P_{l_3}) \\
P_{1,2} &= 22.2\left(0.036(P_{g_2} - P_{l_2}) + 0.018(P_{g_3} - P_{l_3})\right) \\
P_{1,3} &= 11.1\left(0.018(P_{g_2} - P_{l_2}) + 0.054(P_{g_3} - P_{l_3})\right) \\
P_{2,3} &= 11.1\left(0.018(P_{g_2} - P_{l_2}) - 0.036(P_{g_3} - P_{l_3})\right)
\end{aligned}\right\} x = F(u,d)$$

Stochastic OPF

The problem is:

$$\min_{P_{g_2},P_{g_3}} \bar{C}(P_{g_2},P_{g_3}) = \min_{P_{g_2},P_{g_3}} C_1(-P_{g_2} - P_{g_3} + \underbrace{7.00}_{\bar{P}_{l_1}+\bar{P}_{l_2}+\bar{P}_{l_3}}) + C_2(P_{g_2}) + C_3(P_{g_3})$$

$\forall P_{l_1} \in [0.90, 1.10], \forall P_{l_2} \in [0.90, 1.10], \forall P_{l_3} \in [4.80, 5.20]$

$P_{g_1}^{min} \le -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \le P_{g_1}^{max}$

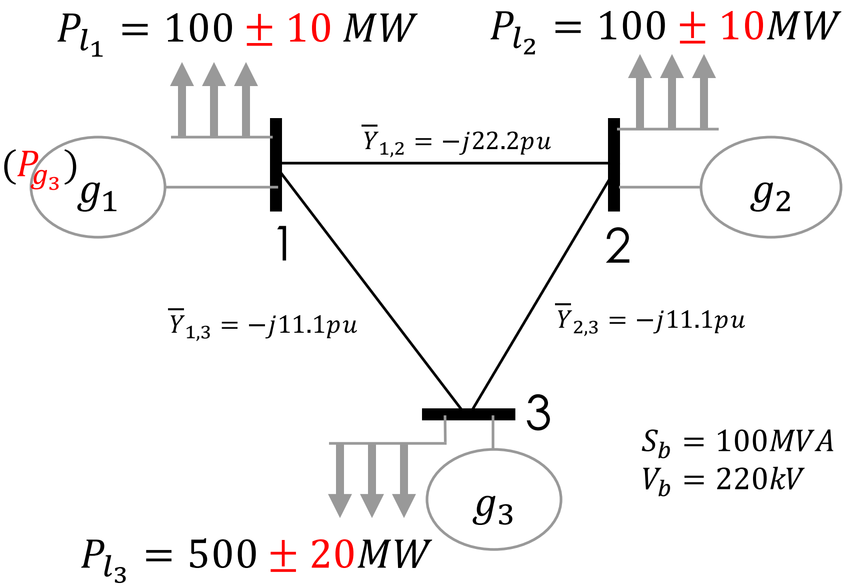
$-P_{1,2}^{max} \le 22.2\left(0.036(P_{g_2} - P_{l_2}) + 0.018(P_{g_3} - P_{l_3})\right) \le P_{1,2}^{max}$

$-P_{1,3}^{max} \le 11.1\left(0.018(P_{g_2} - P_{l_2}) + 0.054(P_{g_3} - P_{l_3})\right) \le P_{1,3}^{max}$

$-P_{2,3}^{max} \le 11.1\left(0.018(P_{g_2} - P_{l_2}) - 0.036(P_{g_3} - P_{l_3})\right) \le P_{2,3}^{max}$

$P_{g_2}^{min} \le P_{g_2} \le P_{g_2}^{max}$

$P_{g_3}^{min} \le P_{g_3} \le P_{g_3}^{max}$



So, we have obtained a robust problem with separable constraints:

$$\min_{u} \bar{C}(u)$$
$$\text{s.t.}\ (a^j)^T u + (b^j)^T d \le \gamma_j, j = 1:J$$
$$\forall d\ \text{s.t.}\ Dd \le c$$

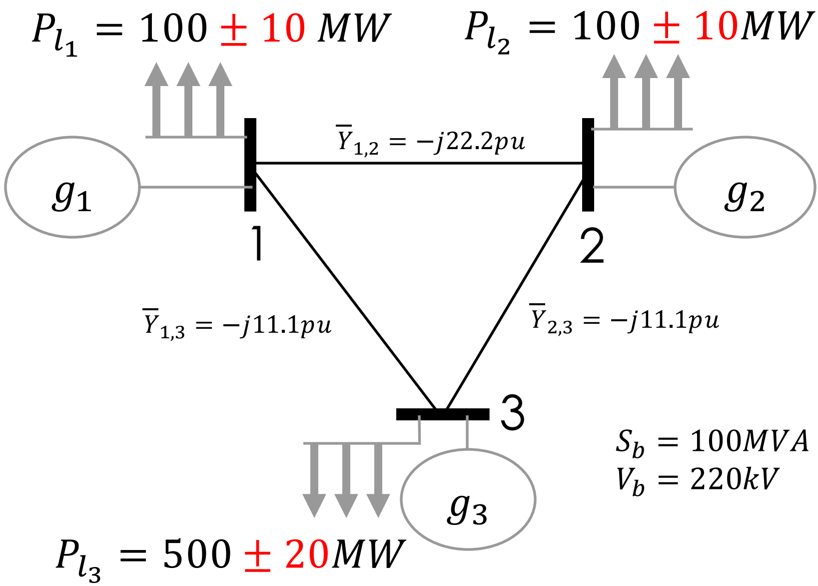
where, **for every** j, we have that a^j, b^j are vectors, with $u = (P_{g_1}, P_{g_2})$ and $d = (P_{l_1}, P_{l_2}, P_{l_3})$. **This problem can be solved using the approach in slide 18 (i.e. solution of robust problems with separable constraints).**

Stochastic OPF

Let's **apply the solution of robust problems with separable constraints to the first one** (assuming $P_{g_1}^{min} = 0$):

$\forall P_{l_1} \in [0.90,1.10], \forall P_{l_2} \in [0.90,1.10], \forall P_{l_3} \in [4.80,5.20]$

$0 \leq -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3}$



That is equivalent to:

$\forall P_{l_1} \in [0.9,1.10], \forall P_{l_2} \in [0.90,1.10], \forall P_{l_3} \in [4.80,5.20]pu$

$P_{g_2} + P_{g_3} \leq P_{l_1} + P_{l_2} + P_{l_3}$

That is equivalent to:

$P_{g_2} + P_{g_3} \leq \min_{P_{l_1}, P_{l_2}, P_{l_3}} (P_{l_1} + P_{l_2} + P_{l_3})$ s.t. $P_{l_1}, P_{l_2} \in [0.90,1.10], P_{l_3} \in [4.80,5.20]$

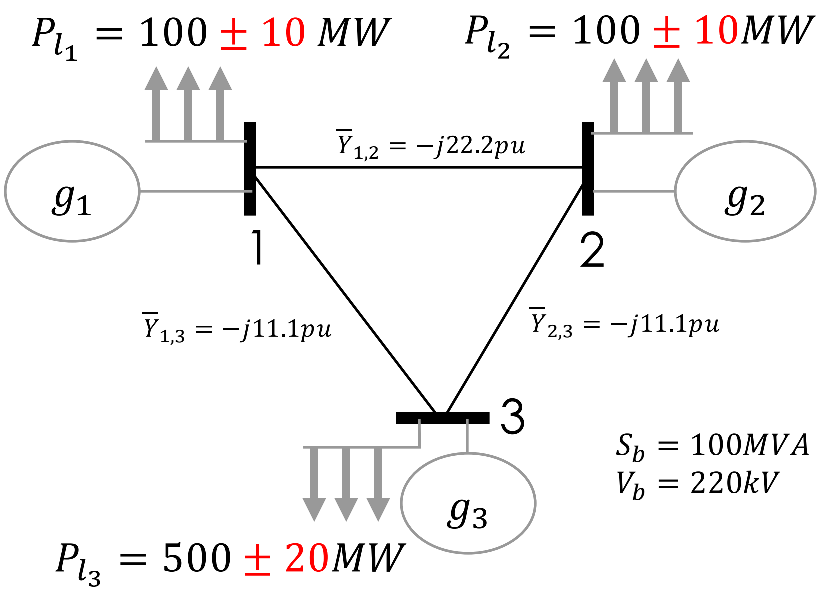
The solution of the above problem gives $h_1 = 0.90 + 0.90 + 4.80 = 6.60$
Hence, the first constraint of the original problem is equivalent to

$$P_{g_2} + P_{g_3} \leq 6.60.$$

This process has to be applied to all the constraints of the original problem.

Stochastic OPF

After having applied the previous process to all the constraints, the original problem becomes:



$$\min_{P_{g_2},P_{g_3}} \bar{C}(P_{g_2}, P_{g_3}) = C_1(-P_{g_2} - P_{g_3} + 7.00) + C_2(P_{g_2}) + C_3(P_{g_3})$$

$s.t.$

$3.4 \le P_{g_2} + P_{g_3} \le 6.6$

$0.96 \le 0.8P_{g_2} + 0.4P_{g_3} \le 4.64$

$0.34 \le 0.2P_{g_2} + 0.6P_{g_3} \le 6.06$

$-0.1 \le -0.2P_{g_2} + 0.4P_{g_3} \le 3.7$

$0 \le P_{g_2} \le 4; 0 \le P_{g_3} \le 4$

The optimal values of the decision variables are:

$$P_{g_2}^* = 2.43pu = 243MW$$

$$P_{g_3}^* = 0.97pu = 97MW$$

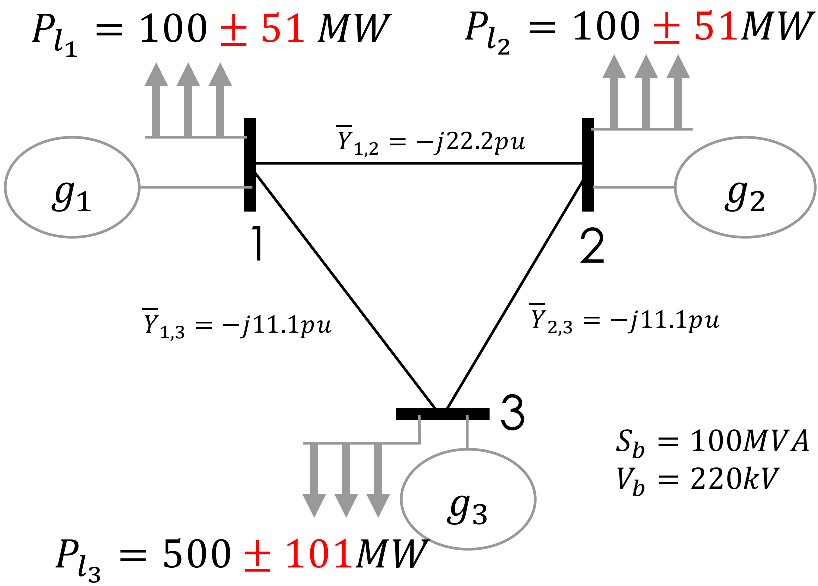
the expected generation $\bar{P}_{g_1}$ from g_1 is:

$$\bar{P}_{g_1} = -P_{g_2} - P_{g_3} + \bar{P}_{l_1} + \bar{P}_{l_2} + \bar{P}_{l_3} = -2.43 - 0.97 + 1 + 1 + 5 = 3.6pu = 360MW$$

and the expected cost is $\bar{C}(P_{g_2}, P_{g_3}) = 27'393CHF$.

Stochastic OPF

It is interesting to note that, **if we increase the uncertainties to values as in the figure**, the problem becomes



$$\min_{P_{g_2},P_{g_3}} \bar{C}(P_{g_2}, P_{g_3}) = C_1(-P_{g_2} - P_{g_3} + 7.00) + C_2(P_{g_2}) + C_3(P_{g_3})$$

$s.t.$

$$5.03 \leq P_{g_2} + P_{g_3} \leq 4.97$$

$$1.612 \leq 0.8P_{g_2} + 0.4P_{g_3} \leq 3.988$$

$$0.908 \leq 0.2P_{g_2} + 0.6P_{g_3} \leq 5.492$$

$$0.306 \leq -0.2P_{g_2} + 0.4P_{g_3} \leq 3.294$$

$$0 \leq P_{g_2} \leq 4; 0 \leq P_{g_3} \leq 4$$

That is **infeasible**. In other words, **the amount of uncertainties in the loads is so high that we cannot find an optimal solution.**

It is also interesting to note that, by increasing the uncertainties, the global cost of the system progressively increases until the problem becomes infeasible.

Stochastic OPF

In general, it is difficult to have deterministic bounds on the stochastic variables (in our case $P_{l_1}, P_{l_2}, P_{l_3}$).
Therefore, we can assume these quantities to be within some bounds with a certain probability.
Let us assume that the **unknown loads are independent** and **follow normal distributions**: $P_{l_i} \sim \mathcal{N}(\bar{P}_{l_i}, v_{l_i})$ where $\bar{P}_{l_i}$ is the expected value and v_{l_i} the variance.

Let η be such that $\mathbb{P}(-\eta \leq X \leq \eta) = 1 - \alpha$ where $X \sim \mathcal{N}(0,1)$ (standard normal distribution, for example, $\eta = 1.96$ when $\alpha = 0.05$).

Then with probability $1 - \varepsilon = (1 - \alpha)^3$:

$$\bar{P}_{l_1} - \eta\sqrt{v_{l_1}} \leq P_{l_1} \leq \bar{P}_{l_1} + \eta\sqrt{v_{l_1}}$$
$$\bar{P}_{l_2} - \eta\sqrt{v_{l_2}} \leq P_{l_2} \leq \bar{P}_{l_2} + \eta\sqrt{v_{l_2}}$$
$$\bar{P}_{l_3} - \eta\sqrt{v_{l_3}} \leq P_{l_3} \leq \bar{P}_{l_3} + \eta\sqrt{v_{l_3}}$$

The above inequalities are called **chance constraints**.

For example, with $\varepsilon = 0.0001$ we must take $\alpha = 3.33 \cdot 10^{-5}$ and $\eta = 4.15$.

Stochastic OPF

With these inequalities

$$\bar{P}_{l_1} - \eta\sqrt{v_{l_1}} \leq P_{l_1} \leq \bar{P}_{l_1} + \eta\sqrt{v_{l_1}}$$
$$\bar{P}_{l_2} - \eta\sqrt{v_{l_2}} \leq P_{l_2} \leq \bar{P}_{l_2} + \eta\sqrt{v_{l_2}}$$
$$\bar{P}_{l_3} - \eta\sqrt{v_{l_3}} \leq P_{l_3} \leq \bar{P}_{l_3} + \eta\sqrt{v_{l_3}}$$

we have now a robust optimization problem, same as before, but with **different bounds on the disturbance and a different cost function (with a term that accounts for the variance).**

Stochastic OPF

To summarize.

System state: $x = F(u, d)$ electric state, power flows.
Decision variables: u generator setpoints $u \in U$.
Disturbance: d **aggregated renewables+loads.**

We want to minimize $\mathbb{E}(C(u, x))$ subject to feasibility ($x \in X$) regardless of disturbance, with high probability

$$\min_{u \in U} \mathbb{E}_d(c(u, F(u, d)))$$
$$s.t. \mathbb{P}(F(u, d) \in X) \geq 1 - \varepsilon$$

where ε is a small probability (risk of failure). The chance constraint is transformed into the constraint $d \in D$ such that $\mathbb{P}(d \in D) = 1 - \varepsilon$:

$$\min_{u \in U} \bar{c}(u)$$
$$s.t. \forall d \in D \; Au + Bd \leq e$$

where $\bar{C}(u) = \mathbb{E}_d(C(u, F(u, d)))$. The problem is then transformed into a standard optimization by eliminating d (i.e., removing the quantifier $\forall$).

Outline

Quantifiers in constraints

Robust problems with separable constraints

Stochastic OPF

Robust OPF

Robust OPF

In power systems, there are many cases where **the grid operator may take decisions that are not necessarily optimal but robust in the sense that the solution is determined for the worst-case cost associated to stochastic variables**.

This is an alternative to stochastic optimization and is also called **non-probabilistic robust optimization**.

We choose a control $u \in U$.

Disturbance $d \in D$ are random.

The system state x becomes $x = F(u, d)$ and we want it to be feasible $(x \in X)$.

We pay a cost $C(u, x) = C(u, F(u, d))$.

Non-probabilistic robust optimization assumes the disturbance is drawn in a set D (no distribution is assumed, only bounds) and the optimization function is the worst-case cost. The constraints are the standard feasibility ones. In other words, we obtain:

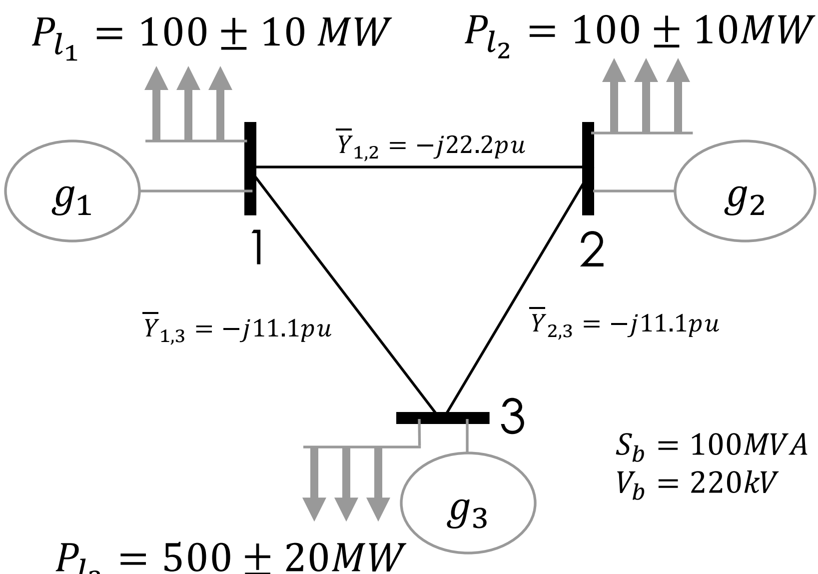
$$\min_{u \in U} \left[\max_{d \in D} C\big(u, F(u, d)\big) \right]$$

$$\text{s.c. } F(u, d) \in X, \ \forall\, d \in D$$

Robust OPF

Let us write the robust OPF for the three-bus example whose load flow constraint are represented by the DC load flow.

Important observation: the constraints are the same as in the stochastic OPF only the objective function is different.



$$\min_{P_{g2},P_{g3}} \left[\max_{\substack{P_{l_1}\in[0.9,1.1],P_{l_2}\in[0.9,1.1] \\ P_{l_3}\in[4.8,5.2]}} \left(C_1\left(-P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3}\right) + C_2 P_{g2} + C_3 P_{g3} \right) \right]$$

$s.t.$

$$\forall P_{l_1} \in [0.90,1.10], \forall P_{l_2} \in [0.90,1.10], \forall P_{l_3} \in [4.80,5.20]$$

$$P_{g_1}^{min} \leq -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \leq P_{g_1}^{max}$$

$$-P_{1,2}^{max} \leq 22.2 \left(0.036\left(P_{g_2} - P_{l_2}\right) + 0.018\left(P_{g_3} - P_{l_3}\right)\right) \leq P_{1,2}^{max}$$

$$-P_{1,3}^{max} \leq 11.1 \left(0.018\left(P_{g_2} - P_{l_2}\right) + 0.054\left(P_{g_3} - P_{l_3}\right)\right) \leq P_{1,3}^{max}$$

$$-P_{2,3}^{max} \leq 11.1 \left(0.018\left(P_{g_2} - P_{l_2}\right) - 0.036\left(P_{g_3} - P_{l_3}\right)\right) \leq P_{2,3}^{max}$$

$$P_{g_2}^{min} \leq P_{g_2} \leq P_{g_2}^{max}$$

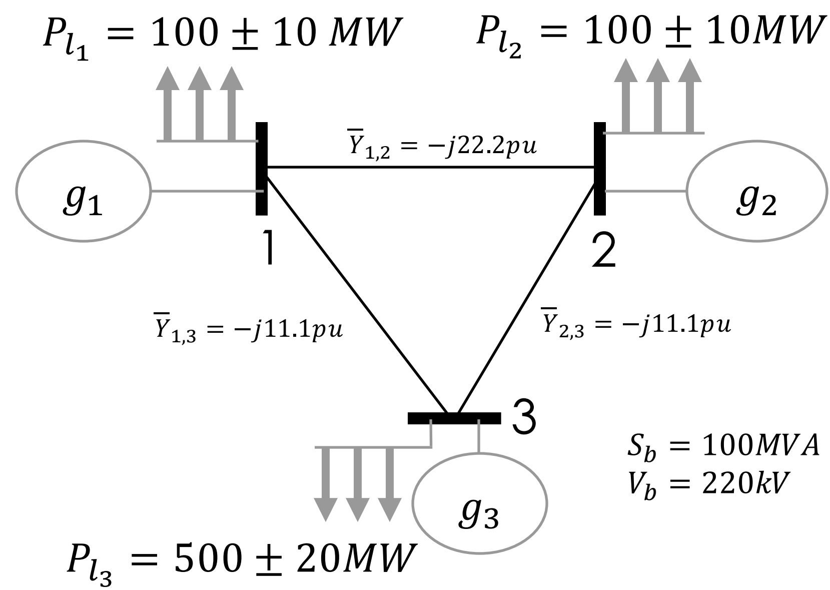
$$P_{g_3}^{min} \leq P_{g_3} \leq P_{g_3}^{max}$$

Robust OPF

For the **inner** max **problem**

$$\max_{\substack{P_{l_1}\in[0.9,1.1],P_{l_2}\in[0.9,1.1]\\ P_{l_3}\in[4.8,5.2]}}\begin{pmatrix} C_1(-P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3}) \\ +C_2P_{g2}+C_3P_{g3}\end{pmatrix}$$

we can easily compute the maximum value (i.e., in **correspondence of the maximum loads**) and obtain the following problem:



$$\min_{P_{g2},P_{g3}}\left(C_1(-P_{g_2}-P_{g_3}+7.4)+C_2P_{g2}+C_3P_{g3}\right)$$

$s.t.$

$$\forall P_{l_1}\in[0.90,1.10],\forall P_{l_2}\in[0.90,1.10],\forall P_{l_3}\in[4.80,5.20]$$

$$P_{g_1}^{min}\le -P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3}\le P_{g_1}^{max}$$

$$-P_{1,2}^{max}\le 22.2\left(0.036(P_{g_2}-P_{l_2})+0.018(P_{g_3}-P_{l_3})\right)\le P_{1,2}^{max}$$

$$-P_{1,3}^{max}\le 11.1\left(0.018(P_{g_2}-P_{l_2})+0.054(P_{g_3}-P_{l_3})\right)\le P_{1,3}^{max}$$

$$-P_{2,3}^{max}\le 11.1\left(0.018(P_{g_2}-P_{l_2})-0.036(P_{g_3}-P_{l_3})\right)\le P_{2,3}^{max}$$

$$P_{g_2}^{min}\le P_{g_2}\le P_{g_2}^{max}$$

$$P_{g_3}^{min}\le P_{g_3}\le P_{g_3}^{max}$$

Observation: the objective of this problem is similar to the one of the stochastic OPF where, instead, the disturbance was taken to the average value:

$$C_1(-P_{g_2}-P_{g_3}+7.0)+C_2(P_{g_2})+C_3(P_{g_3})$$

Here is the worst-case cost.

Robust OPF

Let us try to generalise the removal of max from the problem objective. Robust OPF have the following generic form:

$$\min_{u \in U} \left[\max_{d' \in D} \varphi(u, d') \right]$$
$$\text{s.t.} \left(a^j\right)^T u + \left(b^j\right)^T d \leq \gamma_j, \forall d \in D, j = 1:J$$

To get rid of the max in the objective function, recall from lecture 4.1, the max-removal transformation applied to the above. We have that

$$\min_{u \in U} \left[\max_{j=1:J} \varphi_j(u) \right]$$
$$\text{s.t.}$$
$$(\text{constraints on } u)$$

is equivalent to:

$$\min_{u \in U, \varepsilon \in \mathbb{R}} \varepsilon$$
$$\text{s.t.} \begin{cases} \varepsilon \geq \varphi_j(u), j = 1:J \\ (\text{constraints on } u) \end{cases}$$

Robust OPF

Therefore, our robust OPF of the generic form:

$$\min_{u \in U} \left[\max_{d' \in D} \varphi(u, d') \right]$$
$$\text{s.t.} \left(a^j\right)^T u + \left(b^j\right)^T d \leq \gamma_j, \forall d \in D, j = 1{:}J$$

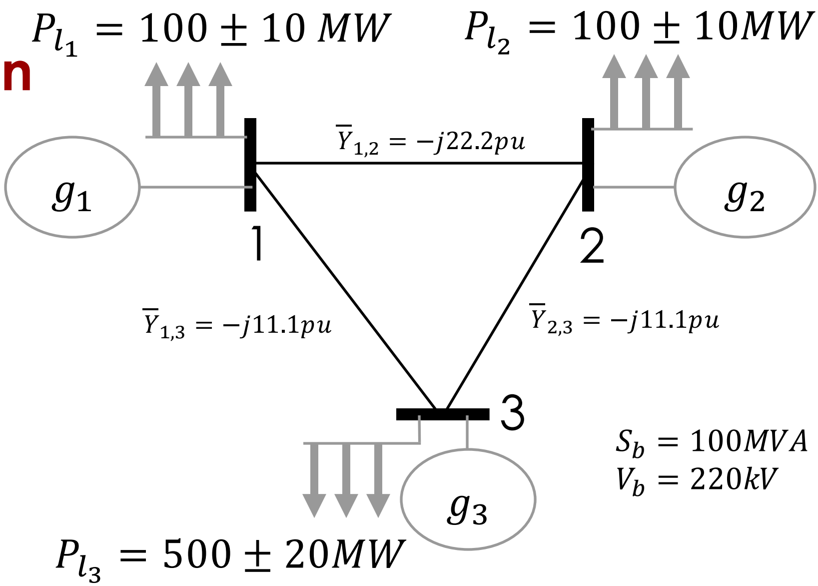
is equivalent to:

$$\min_{u \in U, \varepsilon \in \mathbb{R}} \varepsilon$$
$$\text{s.t.} \begin{cases} \varphi(u, d) \leq \varepsilon, \forall d \in D \\ \left(a^j\right)^T u + \left(b^j\right)^T d \leq \gamma_j, \forall d \in D, j = 1{:}J \end{cases}$$

Note that in the above problem, we have the quantifier ∀ in the constraints, which we can be solved using the approach in slide 18 (i.e. **solution of robust problems with separable constraints**).

Robust OPF

Let us take back our example.
By using the **max-removal transformation we have that the robust OPF is**



$$\min_{P_{g2}, P_{g3}, \varepsilon} \ \varepsilon$$

$s.t.$

$$\forall P_{l_1} \in [0.90, 1.10], \forall P_{l_2} \in [0.90, 1.10], \forall P_{l_3} \in [4.80, 5.20]$$

$$C_1\left(-P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3}\right) + C_2 P_{g2} + C_3 P_{g3} \le \varepsilon$$

$$P_{g_1}^{min} \le -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \le P_{g_1}^{max}$$

$$-P_{1,2}^{max} \le 22.2\left(0.036\left(P_{g_2} - P_{l_2}\right) + 0.018\left(P_{g_3} - P_{l_3}\right)\right) \le P_{1,2}^{max}$$

$$-P_{1,3}^{max} \le 11.1\left(0.018\left(P_{g_2} - P_{l_2}\right) + 0.054\left(P_{g_3} - P_{l_3}\right)\right) \le P_{1,3}^{max}$$

$$-P_{2,3}^{max} \le 11.1\left(0.018\left(P_{g_2} - P_{l_2}\right) - 0.036\left(P_{g_3} - P_{l_3}\right)\right) \le P_{2,3}^{max}$$

$$P_{g_2}^{min} \le P_{g_2} \le P_{g_2}^{max}$$

$$P_{g_3}^{min} \le P_{g_3} \le P_{g_3}^{max}$$

To solve it, we remove ∀, as shown earlier, by computing the worst case over P_{l_i} for each constraint.

Robust OPF

With respect to the robust OPF:

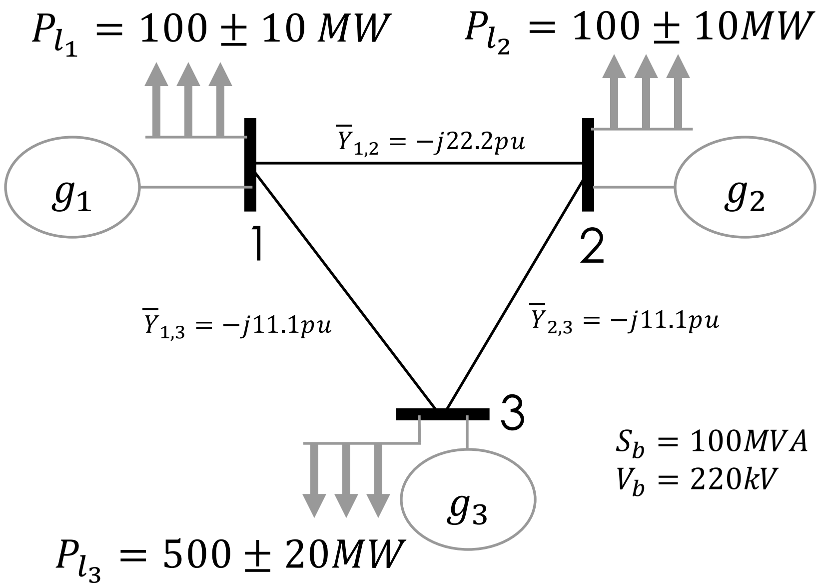
$$\min_{P_{g_2},P_{g_3}}\left[\max_{\substack{P_{l_1}\in[0.9,1.1],P_{l_2}\in[0.9,1.1]\\ P_{l_3}\in[4.8,5.2]}}\left(C_1(-P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3})+C_2P_{g2}+C_3P_{g3}\right)\right]$$

$s.t.$

$$\forall P_{l_1}\in[0.90,1.10],\forall P_{l_2}\in[0.90,1.10],\forall P_{l_3}\in[4.80,5.20]$$
$$P_{g_1}^{min}\le -P_{g_2}-P_{g_3}+P_{l_1}+P_{l_2}+P_{l_3}\le P_{g_1}^{max}$$
$$-P_{1,2}^{max}\le 22.2\left(0.036(P_{g_2}-P_{l_2})+0.018(P_{g_3}-P_{l_3})\right)\le P_{1,2}^{max}$$
$$-P_{1,3}^{max}\le 11.1\left(0.018(P_{g_2}-P_{l_2})+0.054(P_{g_3}-P_{l_3})\right)\le P_{1,3}^{max}$$
$$-P_{2,3}^{max}\le 11.1\left(0.018(P_{g_2}-P_{l_2})-0.036(P_{g_3}-P_{l_3})\right)\le P_{2,3}^{max}$$
$$P_{g_2}^{min}\le P_{g_2}\le P_{g_2}^{max}$$
$$P_{g_3}^{min}\le P_{g_3}\le P_{g_3}^{max}$$

Which formulation is correct ?

1. A
2. B
3. Both
4. None
5. I don't know



(A) $\min_{P_{g_2},P_{g_3},\varepsilon}\ \varepsilon$ s.t.

$$C_1(-P_{g_2}-P_{g_3}+7.4)+C_2P_{g2}+C_3P_{g3}\le\varepsilon$$
$$3.4\le P_{g_2}+P_{g_3}\le 6.6$$
$$0.96\le 0.8P_{g_2}+0.4P_{g_3}\le 4.64$$
$$0.34\le 0.2P_{g_2}+0.6P_{g_3}\le 6.06$$
$$-0.1\le -0.2P_{g_2}+0.4P_{g_3}\le 3.7$$
$$0\le P_{g_2}\le 4;0\le P_{g_3}\le 4$$

(B) $\min_{P_{g_2},P_{g_3}}\ C_1(-P_{g_2}-P_{g_3}+7.4)+C_2P_{g2}+C_3P_{g3}$

s.t.

$$3.4\le P_{g_2}+P_{g_3}\le 6.6$$
$$0.96\le 0.8P_{g_2}+0.4P_{g_3}\le 4.64$$
$$0.34\le 0.2P_{g_2}+0.6P_{g_3}\le 6.06$$
$$-0.1\le -0.2P_{g_2}+0.4P_{g_3}\le 3.7$$
$$0\le P_{g_2}\le 4;0\le P_{g_3}\le 4$$

Robust OPF

With respect to the robust OPF:

$$\min_{P_{g_2},P_{g_3}} \left[\max_{\substack{P_{l_1}\in[0.9,1.1],P_{l_2}\in[0.9,1.1]\\ P_{l_3}\in[4.8,5.2]}} \left(C_1(-P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3}) + C_2 P_{g2} + C_3 P_{g3} \right) \right]$$

$s.t.$

$\forall P_{l_1} \in [0.90, 1.10], \forall P_{l_2} \in [0.90, 1.10], \forall P_{l_3} \in [4.80, 5.20]$

$P_{g_1}^{min} \leq -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \leq P_{g_1}^{max}$

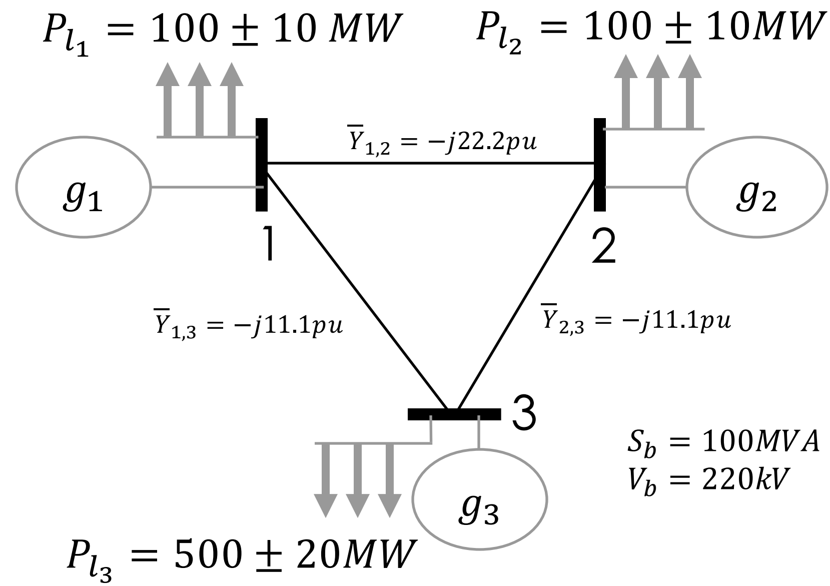
$-P_{1,2}^{max} \leq 22.2 \left(0.036(P_{g_2} - P_{l_2}) + 0.018(P_{g_3} - P_{l_3})\right) \leq P_{1,2}^{max}$

$-P_{1,3}^{max} \leq 11.1 \left(0.018(P_{g_2} - P_{l_2}) + 0.054(P_{g_3} - P_{l_3})\right) \leq P_{1,3}^{max}$

$-P_{2,3}^{max} \leq 11.1 \left(0.018(P_{g_2} - P_{l_2}) - 0.036(P_{g_3} - P_{l_3})\right) \leq P_{2,3}^{max}$

$P_{g_2}^{min} \leq P_{g_2} \leq P_{g_2}^{max}$

$P_{g_3}^{min} \leq P_{g_3} \leq P_{g_3}^{max}$



(A) $\min_{P_{g_2},P_{g_3},\varepsilon} \varepsilon$ s.t.

$C_1(-P_{g_2} - P_{g_3} + 7.4) + C_2 P_{g2} + C_3 P_{g3} \leq \varepsilon$

$3.4 \leq P_{g_2} + P_{g_3} \leq 6.6$

$0.96 \leq 0.8P_{g_2} + 0.4P_{g_3} \leq 4.64$

$0.34 \leq 0.2P_{g_2} + 0.6P_{g_3} \leq 6.06$

$-0.1 \leq -0.2P_{g_2} + 0.4P_{g_3} \leq 3.7$

$0 \leq P_{g_2} \leq 4; 0 \leq P_{g_3} \leq 4$

(B) $\min_{P_{g_2},P_{g_3}} C_1(-P_{g_2} - P_{g_3} + 7.4) + C_2 P_{g2} + C_3 P_{g3}$

s.t.

$3.4 \leq P_{g_2} + P_{g_3} \leq 6.6$

$0.96 \leq 0.8P_{g_2} + 0.4P_{g_3} \leq 4.64$

$0.34 \leq 0.2P_{g_2} + 0.6P_{g_3} \leq 6.06$

$-0.1 \leq -0.2P_{g_2} + 0.4P_{g_3} \leq 3.7$

$0 \leq P_{g_2} \leq 4; 0 \leq P_{g_3} \leq 4$

Which formulation is correct ?

1. A
2. B
3. Both
4. None
5. I don't know

Answer 3

(B) is the formulation seen earlier with the max and is correct.
(A) is the formulation obtained by applying the generic method to remove max from the objective function and is correct.

Both are equivalent. (A) has fewer optimization variables but needs more work. (B) requires less work.

Robust OPF

With respect to the robust OPF:

$$\min_{P_{g_2},P_{g_3}} \left[\max_{\substack{P_{l_1}\in[0.9,1.1],P_{l_2}\in[0.9,1.1] \\ P_{l_3}\in[4.8,5.2]}} \left(C_1(-P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3}) + C_2 P_{g2} + C_3 P_{g3} \right) \right]$$

$s.t.$

$\forall P_{l_1} \in [0.90, 1.10], \forall P_{l_2} \in [0.90, 1.10], \forall P_{l_3} \in [4.80, 5.20]$

$P_{g_1}^{min} \le -P_{g_2} - P_{g_3} + P_{l_1} + P_{l_2} + P_{l_3} \le P_{g_1}^{max}$

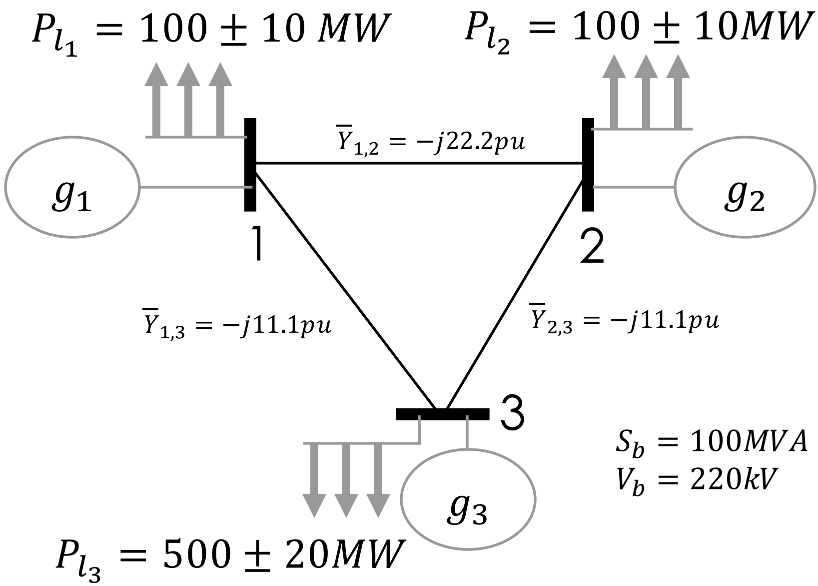
$-P_{1,2}^{max} \le 22.2\left(0.036(P_{g_2} - P_{l_2}) + 0.018(P_{g_3} - P_{l_3})\right) \le P_{1,2}^{max}$

$-P_{1,3}^{max} \le 11.1\left(0.018(P_{g_2} - P_{l_2}) + 0.054(P_{g_3} - P_{l_3})\right) \le P_{1,3}^{max}$

$-P_{2,3}^{max} \le 11.1\left(0.018(P_{g_2} - P_{l_2}) - 0.036(P_{g_3} - P_{l_3})\right) \le P_{2,3}^{max}$

$P_{g_2}^{min} \le P_{g_2} \le P_{g_2}^{max}$

$P_{g_3}^{min} \le P_{g_3} \le P_{g_3}^{max}$



The optimal values of the decision variables are:

$P_{g_2}^* = 2.43pu = 243MW$

$P_{g_3}^* = 0.97pu = 97MW$

So, these are the same obtained for the stochastic OPF.

However, the value of the objective is $27'993CHF$. **Therefore, higher than for the case of the stochastic OPF since we optimised for the worst-case cost.**

(A) $\min_{P_{g2},P_{g3},\varepsilon} \varepsilon$ s.t.

$C_1(-P_{g_2} - P_{g_3} + 7.4) + C_2 P_{g2} + C_3 P_{g3} \le \varepsilon$

$3.4 \le P_{g_2} + P_{g_3} \le 6.6$

$0.96 \le 0.8P_{g_2} + 0.4P_{g_3} \le 4.64$

$0.34 \le 0.2P_{g_2} + 0.6P_{g_3} \le 6.06$

$-0.1 \le -0.2P_{g_2} + 0.4P_{g_3} \le 3.7$

$0 \le P_{g_2} \le 4; 0 \le P_{g_3} \le 4$

(B) $\min_{P_{g2},P_{g3}} C_1(-P_{g_2} - P_{g_3} + 7.4) + C_2 P_{g2} + C_3 P_{g3}$

s.t.

$3.4 \le P_{g_2} + P_{g_3} \le 6.6$

$0.96 \le 0.8P_{g_2} + 0.4P_{g_3} \le 4.64$

$0.34 \le 0.2P_{g_2} + 0.6P_{g_3} \le 6.06$

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$0 \le P_{g_2} \le 4; 0 \le P_{g_3} \le 4$